线性代数
第三章：向量空间
习 题 解 答

宿州学院　数学与统计学院

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1.解

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$$\gamma_1 = 3(2\alpha_1+\alpha_2-5\alpha_3)-(\alpha_1+3\alpha_2+\alpha_3)+(-\alpha_1+4\alpha_2-\alpha_3)$$

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$$\begin{aligned}\gamma_1 &= 3(2\alpha_1+\alpha_2-5\alpha_3)-(\alpha_1+3\alpha_2+\alpha_3)+(-\alpha_1+4\alpha_2-\alpha_3)\\ &= 4\alpha_1+4\alpha_2-17\alpha_3,\end{aligned}$$

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$$\begin{aligned}&\beta_1+\beta_2+\beta_3+\beta_4=3(\alpha_1+\alpha_2+\alpha_3+\alpha_4),\\ &\alpha_1+\alpha_2+\alpha_3+\alpha_4=\frac{1}{3}(\beta_1+\beta_2+\beta_3+\beta_4)\end{aligned},$$

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即$\alpha_1,\alpha_2,\alpha_3,\alpha_4$可以由线性表出$\beta_1,\beta_2,\beta_3,\beta_4$，所以它们等价.

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$$\begin{aligned}
\alpha_1 &= \frac{1}{3}(\beta_1+\beta_2+\beta_3+\beta_4)-\beta_1 = -\frac{2}{3}\beta_1+\frac{1}{3}\beta_2+\frac{1}{3}\beta_3+\frac{1}{3}\beta_4,\\
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因为$(\alpha_1-\alpha_2)+(\alpha_2-\alpha_3)+(\alpha_3-\alpha_1)=0$，

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再讨论$\alpha_1-\alpha_2,\alpha_2-\alpha_3$的线性相关性,

考虑组合$x_1(\alpha_1-\alpha_2)+x_2(\alpha_2-\alpha_3)=0$,

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$$\begin{cases} x_1=0 \\ -x_1+x_2=0 \\ -x_2=0 \end{cases} \Rightarrow \begin{cases} x_1=0 \\ x_2=0 \end{cases} \Rightarrow \alpha_1-\alpha_2,\alpha_2-\alpha_3\text{线性无}$$

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$x_1\alpha_1+x_2(2\alpha_2)+x_3(3\alpha_3)=0$，

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由于$\alpha_1,\alpha_2,\alpha_3$线性相关，所以存在不全为零的数$k_1,k_2,k_3$，使得$k_1\alpha_1+k_2\alpha_2+k_3\alpha_3=0$成立.

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$x_1\alpha_1+x_2(2\alpha_2)+x_3(3\alpha_3)=$

$k_1\alpha_1+(\frac{1}{2}k_2)(2\alpha_2)+(\frac{1}{3}k_3)(3\alpha_3)=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3=0.$

习题3.4($P_{118}-P_{119}$)

$$\begin{cases} x_1=0 \\ -x_1+x_2=0 \\ -x_2=0 \end{cases} \Rightarrow \begin{cases} x_1=0 \\ x_2=0 \end{cases} \Rightarrow \alpha_1-\alpha_2,\alpha_2-\alpha_3$$线性无关，所以$\alpha_1-\alpha_2,\alpha_2-\alpha_3$为其极大相性无关组.

5.解 先考虑向量组$\alpha_1,2\alpha_2,3\alpha_3$的线性相关性.考虑组合

$x_1\alpha_1+x_2(2\alpha_2)+x_3(3\alpha_3)=0$，

由于$\alpha_1,\alpha_2,\alpha_3$线性相关，所以存在不全为零的数$k_1,k_2,k_3$，使得$k_1\alpha_1+k_2\alpha_2+k_3\alpha_3=0$成立.

取$x_1=k_1,x_2=\frac{1}{2}k_2,x_3=\frac{1}{3}k_3$，则$x_1,x_2,x_3$不全为零，且

$x_1\alpha_1+x_2(2\alpha_2)+x_3(3\alpha_3)=$
$k_1\alpha_1+(\frac{1}{2}k_2)(2\alpha_2)+(\frac{1}{3}k_3)(3\alpha_3)=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3=0.$

所以$\alpha_1,2\alpha_2,3\alpha_3$线性相关.

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1, 2\alpha_2$的线性相关性.

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1, 2\alpha_2$的线性相关性.考虑
$x_1\alpha_1 + x_2(2\alpha_2) = x_1\alpha_1 + (2x_2)\alpha_2 = 0$,

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1, 2\alpha_2$的线性相关性.考虑

$$x_1\alpha_1+x_2(2\alpha_2)=x_1\alpha_1+(2x_2)\alpha_2=0,$$

由于α_1,α_2的线性无关，所以

$$x_1\alpha_1+(2x_2)\alpha_2=0 \Leftrightarrow \begin{cases} x_1=0 \\ 2x_2=0 \end{cases}$$

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1,2\alpha_2$的线性相关性.考虑

$$x_1\alpha_1+x_2(2\alpha_2)=x_1\alpha_1+(2x_2)\alpha_2=0,$$

由于α_1,α_2的线性无关，所以

$$x_1\alpha_1+(2x_2)\alpha_2=0\Leftrightarrow\begin{cases}x_1=0\\2x_2=0\end{cases}\Rightarrow\begin{cases}x_1=0\\x_2=0\end{cases},$$

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1, 2\alpha_2$的线性相关性.考虑

$$x_1\alpha_1 + x_2(2\alpha_2) = x_1\alpha_1 + (2x_2)\alpha_2 = 0,$$

由于α_1, α_2的线性无关，所以

$$x_1\alpha_1 + (2x_2)\alpha_2 = 0 \Leftrightarrow \begin{cases} x_1 = 0 \\ 2x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases},$$

所以$\alpha_1, 2\alpha_2$的线性无关.

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1, 2\alpha_2$的线性相关性.考虑

$$x_1\alpha_1 + x_2(2\alpha_2) = x_1\alpha_1 + (2x_2)\alpha_2 = 0,$$

由于α_1, α_2的线性无关，所以

$$x_1\alpha_1 + (2x_2)\alpha_2 = 0 \Leftrightarrow \begin{cases} x_1 = 0 \\ 2x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases},$$

所以$\alpha_1, 2\alpha_2$的线性无关.

$\alpha_1, 2\alpha_2$是$\alpha_1, 2\alpha_2, 3\alpha_3$的极大线性无关组.

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1, 2\alpha_2$的线性相关性.考虑

$$x_1\alpha_1+x_2(2\alpha_2)=x_1\alpha_1+(2x_2)\alpha_2=0,$$

由于α_1,α_2的线性无关，所以

$$x_1\alpha_1+(2x_2)\alpha_2=0\Leftrightarrow\left\{\begin{array}{l}x_1=0\\2x_2=0\end{array}\right.\Rightarrow\left\{\begin{array}{l}x_1=0\\x_2=0\end{array}\right.,$$

所以$\alpha_1, 2\alpha_2$的线性无关.

$\alpha_1, 2\alpha_2$是$\alpha_1, 2\alpha_2, 3\alpha_3$的极大线性无关组.

6.解

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1, 2\alpha_2$的线性相关性.考虑

$$x_1\alpha_1 + x_2(2\alpha_2) = x_1\alpha_1 + (2x_2)\alpha_2 = 0,$$

由于α_1, α_2的线性无关，所以

$$x_1\alpha_1 + (2x_2)\alpha_2 = 0 \Leftrightarrow \begin{cases} x_1 = 0 \\ 2x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases},$$

所以$\alpha_1, 2\alpha_2$的线性无关.

$\alpha_1, 2\alpha_2$是$\alpha_1, 2\alpha_2, 3\alpha_3$的极大线性无关组.

6.解 因为β, γ, δ线性无关，所以它的部分组β, γ也线性无关.

习题3.4($P_{118}-P_{119}$)

再考虑$\alpha_1, 2\alpha_2$的线性相关性.考虑

$$x_1\alpha_1 + x_2(2\alpha_2) = x_1\alpha_1 + (2x_2)\alpha_2 = 0,$$

由于α_1, α_2的线性无关，所以

$$x_1\alpha_1 + (2x_2)\alpha_2 = 0 \Leftrightarrow \begin{cases} x_1 = 0 \\ 2x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases},$$

所以$\alpha_1, 2\alpha_2$的线性无关.

$\alpha_1, 2\alpha_2$是$\alpha_1, 2\alpha_2, 3\alpha_3$的极大线性无关组.

6.解 因为β, γ, δ线性无关，所以它的部分组β, γ也线性无关.

而已知α, β, γ线性相关，所以β, γ是向量组α, β, γ的极大线性无关组.

习题3.4($P_{118}-P_{119}$)

7.证明

习题3.4($P_{118}-P_{119}$)

7.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$是向量组$\beta_1,\beta_2,\ldots,\beta_t$是一个极大线性无关组.

习题3.4($P_{118}-P_{119}$)

7.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$是向量组$\beta_1,\beta_2,\ldots,\beta_t$是一个极大线性无关组.

由于极大线性无关组与向量组自身等价，

所以 $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出；

$\beta_1,\beta_2,\ldots,\beta_t$可以由$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

习题3.4($P_{118}-P_{119}$)

7.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$是向量组$\beta_1,\beta_2,\ldots,\beta_t$是一个极大线性无关组.

由于极大线性无关组与向量组自身等价，

所以 $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出；

$\beta_1,\beta_2,\ldots,\beta_t$可以由$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

而已知$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\beta_1,\beta_2,\ldots,\beta_t$线性表出，再由向量组线性表出的传递性，

则有$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由 $\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

习题3.4($P_{118}-P_{119}$)

7.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$是向量组$\beta_1,\beta_2,\ldots,\beta_t$是一个极大线性无关组.

由于极大线性无关组与向量组自身等价，

所以 $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出；

$\beta_1,\beta_2,\ldots,\beta_t$可以由$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

而已知$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\beta_1,\beta_2,\ldots,\beta_t$线性表出，再由向量组线性表出的传递性，

则有$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由 $\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

又因为$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$线性无关，所以$m\le n$.

习题3.4($P_{118}-P_{119}$)

7.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$是向量组$\beta_1,\beta_2,\ldots,\beta_t$是一个极大线性无关组.

由于极大线性无关组与向量组自身等价，

所以 $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出；

$\beta_1,\beta_2,\ldots,\beta_t$可以由$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

而已知$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\beta_1,\beta_2,\ldots,\beta_t$线性表出，再由向量组线性表出的传递性，

则有$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由 $\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

又因为$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$线性无关，所以$m\leq n$.

而m为向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$的秩，

n为向量组$\beta_1,\beta_2,\ldots,\beta_t$的秩，

习题3.4($P_{118}-P_{119}$)

7.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$是向量组$\beta_1,\beta_2,\ldots,\beta_t$是一个极大线性无关组.

由于极大线性无关组与向量组自身等价，

所以 $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出；

$\beta_1,\beta_2,\ldots,\beta_t$可以由$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

而已知$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\beta_1,\beta_2,\ldots,\beta_t$线性表出，再由向量组线性表出的传递性，

则有$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$可以由 $\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}$线性表出.

又因为$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}$线性无关，所以$m\leq n$.

而m为向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$的秩，

n为向量组$\beta_1,\beta_2,\ldots,\beta_t$的秩，

所以$\alpha_1,\alpha_2,\ldots,\alpha_s$的秩$\leq\beta_1,\beta_2,\ldots,\beta_t$的秩.

习题3.4($P_{118} - P_{119}$)

8.证明

习题3.4($P_{118}-P_{119}$)

8.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$是向量组$\beta_1,\beta_2,\ldots,\beta_s$是一个极大线性无关组.

习题3.4($P_{118}-P_{119}$)

8.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$是向量组$\beta_1,\beta_2,\ldots,\beta_s$是一个极大线性无关组. 由于极大线性无关组与向量组自身等价，所以$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$线性表出；$\beta_1,\beta_2,\ldots,\beta_s$可以由线性表出$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$.

习题3.4($P_{118}-P_{119}$)

8.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$是向量组$\beta_1,\beta_2,\ldots,\beta_s$是一个极大线性无关组. 由于极大线性无关组与向量组自身等价，所以$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$线性表出；$\beta_1,\beta_2,\ldots,\beta_s$可以由线性表出$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$.

从而向量组

$\alpha_1,\ldots,\alpha_s,\beta_1,\ldots,\beta_s$可以由 $\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$线性表出.

习题3.4($P_{118}-P_{119}$)

8.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$是向量组$\beta_1,\beta_2,\ldots,\beta_s$是一个极大线性无关组. 由于极大线性无关组与向量组自身等价，所以$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$线性表出；$\beta_1,\beta_2,\ldots,\beta_s$可以由线性表出$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$.

从而向量组

$\alpha_1,\ldots,\alpha_s,\beta_1,\ldots,\beta_s$可以由 $\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$线性表出. 又因为$\alpha_1+\beta_1,\alpha_2+\beta_2,\ldots,\alpha_s+\beta_s$可以由$\alpha_1,\ldots,\alpha_s,\beta_1,\ldots,\beta_s$线性表出，利用线性表出的传递性，$\alpha_1+\beta_1,\alpha_2+\beta_2,\ldots,\alpha_s+\beta_s$可以由$\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$线性表出.

习题3.4($P_{118}-P_{119}$)

8.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$是向量组$\beta_1,\beta_2,\ldots,\beta_s$是一个极大线性无关组. 由于极大线性无关组与向量组自身等价，所以$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$线性表出；$\beta_1,\beta_2,\ldots,\beta_s$可以由线性表出$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$.

从而向量组

$\alpha_1,\ldots,\alpha_s,\beta_1,\ldots,\beta_s$可以由 $\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$线性表出. 又因为$\alpha_1+\beta_1,\alpha_2+\beta_2,\ldots,\alpha_s+\beta_s$可以由$\alpha_1,\ldots,\alpha_s,\beta_1,\ldots,\beta_s$线性表出，利用线性表出的传递性，$\alpha_1+\beta_1,\alpha_2+\beta_2,\ldots,\alpha_s+\beta_s$可以由$\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$线性表出. 而$\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$的秩不超过其向量个数$r_1+r_2$，

习题3.4($P_{118}-P_{119}$)

8.证明 设$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$是向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$是向量组$\beta_1,\beta_2,\ldots,\beta_s$是一个极大线性无关组. 由于极大线性无关组与向量组自身等价，所以$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_{r_1}}$线性表出；$\beta_1,\beta_2,\ldots,\beta_s$可以由线性表出$\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_{r_2}}$.

从而向量组

$\alpha_1,\ldots,\alpha_s,\beta_1,\ldots,\beta_s$可以由 $\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$线性表出. 又因为$\alpha_1+\beta_1,\alpha_2+\beta_2,\ldots,\alpha_s+\beta_s$可以由$\alpha_1,\ldots,\alpha_s,\beta_1,\ldots,\beta_s$线性表出，利用线性表出的传递性，$\alpha_1+\beta_1,\alpha_2+\beta_2,\ldots,\alpha_s+\beta_s$可以由$\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$线性表出. 而$\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$的秩不超过其向量个数$r_1+r_2$，再由$Ex_7$的结论，$\alpha_1+\beta_1,\alpha_2+\beta_2,\ldots,\alpha_s+\beta_s$的秩$\leq$ $\alpha_{i_1},\ldots,\alpha_{i_{r_1}},\beta_{j_1},\ldots,\beta_{j_{r_2}}$的秩，

习题3.4($P_{118}-P_{119}$)

8.证明 设$\alpha_{i_1},\alpha_{i_2},\dots,\alpha_{i_{r_1}}$是向量组$\alpha_1,\alpha_2,\dots,\alpha_s$是一个极大线性无关组，$\beta_{j_1},\beta_{j_2},\dots,\beta_{j_{r_2}}$是向量组$\beta_1,\beta_2,\dots,\beta_s$是一个极大线性无关组. 由于极大线性无关组与向量组自身等价，所以$\alpha_1,\alpha_2,\dots,\alpha_s$可以由$\alpha_{i_1},\alpha_{i_2},\dots,\alpha_{i_{r_1}}$线性表出；$\beta_1,\beta_2,\dots,\beta_s$可以由线性表出$\beta_{j_1},\beta_{j_2},\dots,\beta_{j_{r_2}}$.

从而向量组

$\alpha_1,\dots,\alpha_s,\beta_1,\dots,\beta_s$可以由 $\alpha_{i_1},\dots,\alpha_{i_{r_1}},\beta_{j_1},\dots,\beta_{j_{r_2}}$线性表出. 又因为$\alpha_1+\beta_1,\alpha_2+\beta_2,\dots,\alpha_s+\beta_s$可以由$\alpha_1,\dots,\alpha_s,\beta_1,\dots,\beta_s$线性表出，利用线性表出的传递性，$\alpha_1+\beta_1,\alpha_2+\beta_2,\dots,\alpha_s+\beta_s$可以由$\alpha_{i_1},\dots,\alpha_{i_{r_1}},\beta_{j_1},\dots,\beta_{j_{r_2}}$线性表出. 而$\alpha_{i_1},\dots,\alpha_{i_{r_1}},\beta_{j_1},\dots,\beta_{j_{r_2}}$的秩不超过其向量个数$r_1+r_2$，再由$Ex_7$的结论，$\alpha_1+\beta_1,\alpha_2+\beta_2,\dots,\alpha_s+\beta_s$的秩$\leq$ $\alpha_{i_1},\dots,\alpha_{i_{r_1}},\beta_{j_1},\dots,\beta_{j_{r_2}}$的秩，即$r_3\leq r_1+r_2$.

习题3.4($P_{118}-P_{119}$)

9.证明

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

假设任一n维向量都可以由 $\alpha_1,\alpha_2,...,\alpha_n$线性表出，取$n$维规范单位向量组$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$，则$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$可以由$\alpha_1,\alpha_2,...,\alpha_n$线性表出；

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

假设任一n维向量都可以由 $\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出，取$n$维规范单位向量组$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$，则$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$可以由$\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出；而任意一个$n$维向量都可以由$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$线性表出，所以 $\alpha_1,\alpha_2,\ldots,\alpha_n$与$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$等价，

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

假设任一n维向量都可以由 $\alpha_1,\alpha_2,...,\alpha_n$线性表出，取$n$维规范单位向量组$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$，则$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$可以由$\alpha_1,\alpha_2,...,\alpha_n$线性表出；而任意一个$n$维向量都可以由$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$线性表出，所以 $\alpha_1,\alpha_2,...,\alpha_n$与$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$等价，等价向量组有相同的秩，而 $\varepsilon_1,\varepsilon_2,...,\varepsilon_n$的秩为$n$，

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

假设任一n维向量都可以由 $\alpha_1,\alpha_2,...,\alpha_n$线性表出，取$n$维规范单位向量组$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$，则$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$可以由$\alpha_1,\alpha_2,...,\alpha_n$线性表出；而任意一个$n$维向量都可以由$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$线性表出，所以 $\alpha_1,\alpha_2,...,\alpha_n$与$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$等价，等价向量组有相同的秩，而 $\varepsilon_1,\varepsilon_2,...,\varepsilon_n$的秩为$n$，

所以$\alpha_1,\alpha_2,...,\alpha_n$的秩为$n$，$\alpha_1,\alpha_2,...,\alpha_n$线性无关.

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

假设任一n维向量都可以由 $\alpha_1,\alpha_2,...,\alpha_n$线性表出，取$n$维规范单位向量组$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$，则$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$可以由$\alpha_1,\alpha_2,...,\alpha_n$线性表出；而任意一个$n$维向量都可以由$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$线性表出，所以 $\alpha_1,\alpha_2,...,\alpha_n$与$\varepsilon_1,\varepsilon_2,...,\varepsilon_n$等价，等价向量组有相同的秩，而 $\varepsilon_1,\varepsilon_2,...,\varepsilon_n$的秩为$n$，

所以$\alpha_1,\alpha_2,...,\alpha_n$的秩为$n$，$\alpha_1,\alpha_2,...,\alpha_n$线性无关.

必要性.

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

假设任一n维向量都可以由 $\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出，取$n$维规范单位向量组$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$，则$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$可以由$\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出；而任意一个$n$维向量都可以由$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$线性表出，所以 $\alpha_1,\alpha_2,\ldots,\alpha_n$与$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$等价，等价向量组有相同的秩，而 $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$的秩为$n$，

所以$\alpha_1,\alpha_2,\ldots,\alpha_n$的秩为$n$，$\alpha_1,\alpha_2,\ldots,\alpha_n$线性无关.

必要性.假设$\alpha_1,\alpha_2,\ldots,\alpha_n$线性无关，任取一个$n$维向量$\alpha$，则$n+1$个$n$维向量$\alpha_1,\alpha_2,\ldots,\alpha_n,\alpha$线性相关.

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

假设任一n维向量都可以由 $\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出，取$n$维规范单位向量组$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$，则$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$可以由$\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出；而任意一个$n$维向量都可以由$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$线性表出，所以 $\alpha_1,\alpha_2,\ldots,\alpha_n$与$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$等价，等价向量组有相同的秩，而 $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$的秩为$n$，

所以$\alpha_1,\alpha_2,\ldots,\alpha_n$的秩为$n$，$\alpha_1,\alpha_2,\ldots,\alpha_n$线性无关.

必要性.假设$\alpha_1,\alpha_2,\ldots,\alpha_n$线性无关，任取一个$n$维向量$\alpha$，则$n+1$个$n$维向量$\alpha_1,\alpha_2,\ldots,\alpha_n,\alpha$线性相关. 而$\alpha_1,\alpha_2,\ldots,\alpha_n$线性无关，所以$\alpha$可以由$\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出.

习题3.4($P_{118}-P_{119}$)

9.证明 先证明充分性.

假设任一n维向量都可以由 $\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出，取$n$维规范单位向量组$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$，则$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$可以由$\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出；而任意一个$n$维向量都可以由$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$线性表出，所以 $\alpha_1,\alpha_2,\ldots,\alpha_n$与$\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$等价，等价向量组有相同的秩，而 $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$的秩为$n$，

所以$\alpha_1,\alpha_2,\ldots,\alpha_n$的秩为$n$，$\alpha_1,\alpha_2,\ldots,\alpha_n$线性无关.

必要性.假设$\alpha_1,\alpha_2,\ldots,\alpha_n$线性无关，任取一个$n$维向量$\alpha$，则$n+1$个$n$维向量$\alpha_1,\alpha_2,\ldots,\alpha_n,\alpha$线性相关. 而$\alpha_1,\alpha_2,\ldots,\alpha_n$线性无关，所以$\alpha$可以由$\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出.

即，任一n维向量都可以由$\alpha_1,\alpha_2,\ldots,\alpha_n$线性表出.

习题3.4($P_{118}-P_{119}$)

10.证明

习题3.4($P_{118}-P_{119}$)

10.证明 设秩为r的向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由其$r$个向量构成的部分组$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出.

习题3.4($P_{118}-P_{119}$)

10.证明 设秩为r的向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由其$r$个向量构成的部分组$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出.

要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$是其极大线性无关组，只要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$线性无关.

习题3.4($P_{118}-P_{119}$)

10.证明 设秩为r的向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由其$r$个向量构成的部分组$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出.

要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$是其极大线性无关组，只要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$线性无关.

由于$\alpha_1,\alpha_2,\ldots,\alpha_s$的秩为$r$，所以其存在含$r$向量的极大线性无关组，设为$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$.

习题3.4($P_{118}-P_{119}$)

10.证明 设秩为r的向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由其$r$个向量构成的部分组$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出.

要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$是其极大线性无关组，只要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$线性无关.

由于$\alpha_1,\alpha_2,\ldots,\alpha_s$的秩为$r$，所以其存在含$r$向量的极大线性无关组，设为$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$.

则$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出，从而可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出.

习题3.4($P_{118}-P_{119}$)

10.证明 设秩为r的向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由其$r$个向量构成的部分组$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出.

要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$是其极大线性无关组，只要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$线性无关.

由于$\alpha_1,\alpha_2,\ldots,\alpha_s$的秩为$r$，所以其存在含$r$向量的极大线性无关组，设为$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$.

则$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出，从而可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出. 且$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$中的每一个向量都可以由 $\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$线性表出，

所以 $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$与$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$等价，

习题3.4($P_{118}-P_{119}$)

10.证明 设秩为r的向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由其$r$个向量构成的部分组$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出.

要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$是其极大线性无关组，只要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$线性无关.

由于$\alpha_1,\alpha_2,\ldots,\alpha_s$的秩为$r$，所以其存在含$r$向量的极大线性无关组，设为$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$.

则$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出，从而可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出. 且$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$中的每一个向量都可以由 $\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$线性表出，

所以 $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$与$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$等价，等价向量组有相同的秩，所以$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$的秩为$r$，

习题3.4($P_{118}-P_{119}$)

10.证明 设秩为r的向量组$\alpha_1,\alpha_2,\ldots,\alpha_s$可以由其$r$个向量构成的部分组$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出.

要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$是其极大线性无关组，只要证明$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$线性无关.

由于$\alpha_1,\alpha_2,\ldots,\alpha_s$的秩为$r$，所以其存在含$r$向量的极大线性无关组，设为$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$.

则$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$可以由$\alpha_1,\alpha_2,\ldots,\alpha_s$线性表出，从而可以由$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$ 线性表出. 且$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$中的每一个向量都可以由 $\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$线性表出，

所以 $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$与$\alpha_{j_1},\alpha_{j_2},\ldots,\alpha_{j_r}$等价，等价向量组有相同的秩，所以$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$的秩为$r$，所以$\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_r}$线性无关.

习题3.4($P_{118}-P_{119}$)

11.证明

习题3.4($P_{118}-P_{119}$)

11.证明 设(I')是向量组(I)的极大线性无关组，(II')是向量组(II)的极大线性无关组.

习题3.4($P_{118}-P_{119}$)

11.证明 设(I′)是向量组(I)的极大线性无关组，(II′)是向量组(II)的极大线性无关组.由于向量组(I)与(II)有相同的秩，且(I)可以由(II)表出，所以向量组(I′)可以由向量组(II′)线性表出且它们有相同的向量个数，并设个数为t.

习题3.4($P_{118}-P_{119}$)

11.证明 设(I')是向量组(I)的极大线性无关组，(II')是向量组(II)的极大线性无关组.由于向量组(I)与(II)有相同的秩，且(I)可以由(II)表出，所以向量组(I')可以由向量组(II')线性表出且它们有相同的向量个数，并设个数为t.

任取(II')的一个向量β，由于(I')可以由(II')线性表出且β是(II')中的向量，所以由(I')和β构成的含$t+1$个向量的向量组可以由含t个向量的向量组(II')线性表出.

习题3.4($P_{118}-P_{119}$)

11.证明 设(I')是向量组(I)的极大线性无关组，(II')是向量组(II)的极大线性无关组.由于向量组(I)与(II)有相同的秩，且(I)可以由(II)表出，所以向量组(I')可以由向量组(II')线性表出且它们有相同的向量个数，并设个数为t.

任取(II')的一个向量β，由于(I')可以由(II')线性表出且β是(II')中的向量，所以由(I')和β构成的含$t+1$个向量的向量组可以由含t个向量的向量组(II')线性表出.所以由(I')和β构成的向量组线性相关.

习题3.4($P_{118}-P_{119}$)

11.证明 设(I')是向量组(I)的极大线性无关组，(II')是向量组(II)的极大线性无关组.由于向量组(I)与(II)有相同的秩，且(I)可以由(II)表出，所以向量组(I')可以由向量组(II')线性表出且它们有相同的向量个数，并设个数为t.

任取(II')的一个向量β，由于(I')可以由(II')线性表出且β是(II')中的向量，所以由(I')和β构成的含$t+1$个向量的向量组可以由含t个向量的向量组(II')线性表出.所以由(I')和β构成的向量组线性相关.

而(I')线性无关，所以β可以由(I')线性表出，

习题3.4($P_{118}-P_{119}$)

11.证明 设(I')是向量组(I)的极大线性无关组，(II')是向量组(II)的极大线性无关组.由于向量组(I)与(II)有相同的秩，且(I)可以由(II)表出，所以向量组(I')可以由向量组(II')线性表出且它们有相同的向量个数，并设个数为t.

任取(II')的一个向量β，由于(I')可以由(II')线性表出且β是(II')中的向量，所以由(I')和β构成的含$t+1$个向量的向量组可以由含t个向量的向量组(II')线性表出.所以由(I')和β构成的向量组线性相关.

而(I')线性无关，所以β可以由(I')线性表出，注意到β在(II')中的任意性，则(II')可以由(I')线性表出.

习题3.4($P_{118} - P_{119}$)

11.证明 设(I')是向量组(I)的极大线性无关组，(II')是向量组(II)的极大线性无关组.由于向量组(I)与(II)有相同的秩，且(I)可以由(II)表出，所以向量组(I')可以由向量组(II')线性表出且它们有相同的向量个数，并设个数为t.

任取(II')的一个向量β，由于(I')可以由(II')线性表出且β是(II')中的向量，所以由(I')和β构成的含$t+1$个向量的向量组可以由含t个向量的向量组(II')线性表出.所以由(I')和β构成的向量组线性相关.

而(I')线性无关，所以β可以由(I')线性表出，注意到β在(II')中的任意性，则(II')可以由(I')线性表出.

所以(I')与(II')等价.

习题3.4($P_{118}-P_{119}$)

11.证明 设(I′)是向量组(I)的极大线性无关组，(II′)是向量组(II)的极大线性无关组.由于向量组(I)与(II)有相同的秩，且(I)可以由(II)表出，所以向量组(I′)可以由向量组(II′)线性表出且它们有相同的向量个数，并设个数为t.

任取(II′)的一个向量β，由于(I′)可以由(II′)线性表出且β是(II′)中的向量，所以由(I′)和β构成的含$t+1$个向量的向量组可以由含t个向量的向量组(II′)线性表出.所以由(I′)和β构成的向量组线性相关.

而(I′)线性无关，所以β可以由(I′)线性表出，注意到β在(II′)中的任意性，则(II′)可以由(I′)线性表出.

所以(I′)与(II′)等价.

利用等价的传递性，(I)等价于(I′)，(I′)等价于(II′)，(II′)等价于(II)，

习题3.4($P_{118}-P_{119}$)

11.证明 设(I')是向量组(I)的极大线性无关组，(II')是向量组(II)的极大线性无关组.由于向量组(I)与(II)有相同的秩，且(I)可以由(II)表出，所以向量组(I')可以由向量组(II')线性表出且它们有相同的向量个数，并设个数为t.

任取(II')的一个向量β，由于(I')可以由(II')线性表出且β是(II')中的向量，所以由(I')和β构成的含$t+1$个向量的向量组可以由含t个向量的向量组(II')线性表出.所以由(I')和β构成的向量组线性相关.

而(I')线性无关，所以β可以由(I')线性表出，注意到β在(II')中的任意性，则(II')可以由(I')线性表出.

所以(I')与(II')等价.

利用等价的传递性，(I)等价于(I')，(I')等价于(II')，(II')等价于(II)，所以(I)等价于(II).

习题3.4($P_{118}-P_{119}$)

12.证明

习题3.4($P_{118}-P_{119}$)

12.证明 因为rank(I)=rank(II)=3，所以$\alpha_1,\alpha_2,\alpha_3$线性无关，且$\alpha_1,\alpha_2,\alpha_3,\alpha_4$线性相关.

习题3.4($P_{118}-P_{119}$)

12.证明 因为rank(I)=rank(II)=3，所以$\alpha_1,\alpha_2,\alpha_3$线性无关，且$\alpha_1,\alpha_2,\alpha_3,\alpha_4$线性相关. 所以$\alpha_4$可以由$\alpha_1,\alpha_2,\alpha_3$线性表出，即存在系数$k_1,k_2,k_3$，使得$\alpha_4=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3$.

习题3.4($P_{118}-P_{119}$)

12.证明 因为rank(I)=rank(II)=3，所以$\alpha_1,\alpha_2,\alpha_3$线性无关，且$\alpha_1,\alpha_2,\alpha_3,\alpha_4$线性相关. 所以$\alpha_4$可以由$\alpha_1,\alpha_2,\alpha_3$线性表出，即存在系数$k_1,k_2,k_3$，使得$\alpha_4=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3$.

要证$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$的秩为4，只要证明$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$线性无关.

习题3.4($P_{118}-P_{119}$)

12.证明 因为rank(I)=rank(II)=3，所以$\alpha_1,\alpha_2,\alpha_3$线性无关，且$\alpha_1,\alpha_2,\alpha_3,\alpha_4$线性相关. 所以$\alpha_4$可以由$\alpha_1,\alpha_2,\alpha_3$线性表出，即存在系数$k_1,k_2,k_3$，使得$\alpha_4=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3$.

要证$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$的秩为4，只要证明$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$线性无关.

考虑组合$x_1\alpha_1+x_2\alpha_2+x_3\alpha_3+x_4(\alpha_5-\alpha_4)=0$，即$(x_1-k_1x_4)\alpha_1+(x_2-k_2x_4)\alpha_2+(x_3-k_3x_4)\alpha_3+x_4\alpha_5=0$.

习题3.4($P_{118}-P_{119}$)

12.证明 因为rank(I)=rank(II)=3，所以$\alpha_1,\alpha_2,\alpha_3$线性无关，且$\alpha_1,\alpha_2,\alpha_3,\alpha_4$线性相关. 所以$\alpha_4$可以由$\alpha_1,\alpha_2,\alpha_3$线性表出，即存在系数$k_1,k_2,k_3$，使得$\alpha_4=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3$.

要证$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$的秩为4，只要证明$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$线性无关.

考虑组合$x_1\alpha_1+x_2\alpha_2+x_3\alpha_3+x_4(\alpha_5-\alpha_4)=0$，即$(x_1-k_1x_4)\alpha_1+(x_2-k_2x_4)\alpha_2+(x_3-k_3x_4)\alpha_3+x_4\alpha_5=0$.

由于(III)的秩为4，所以$\alpha_1,\alpha_2,\alpha_3,\alpha_5$线性无关，

习题3.4($P_{118}-P_{119}$)

12.证明 因为rank(I)=rank(II)=3，所以$\alpha_1,\alpha_2,\alpha_3$线性无关，且$\alpha_1,\alpha_2,\alpha_3,\alpha_4$线性相关. 所以$\alpha_4$可以由$\alpha_1,\alpha_2,\alpha_3$线性表出，即存在系数$k_1,k_2,k_3$，使得$\alpha_4=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3$.

要证$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$的秩为4，只要证明$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$线性无关.

考虑组合$x_1\alpha_1+x_2\alpha_2+x_3\alpha_3+x_4(\alpha_5-\alpha_4)=0$，即$(x_1-k_1x_4)\alpha_1+(x_2-k_2x_4)\alpha_2+(x_3-k_3x_4)\alpha_3+x_4\alpha_5=0$.

由于(III)的秩为4，所以$\alpha_1,\alpha_2,\alpha_3,\alpha_5$线性无关，所以

$$\begin{cases} x_1-k_1x_4=0 \\ x_2-k_2x_4=0 \\ x_3-k_3x_4=0 \\ x_4=0 \end{cases} \Rightarrow \begin{cases} x_1=0 \\ x_2=0 \\ x_3=0 \\ x_4=0 \end{cases}$$

习题3.4($P_{118}-P_{119}$)

12.证明 因为rank(I)=rank(II)=3，所以$\alpha_1,\alpha_2,\alpha_3$线性无关，且$\alpha_1,\alpha_2,\alpha_3,\alpha_4$线性相关. 所以$\alpha_4$可以由$\alpha_1,\alpha_2,\alpha_3$线性表出，即存在系数$k_1,k_2,k_3$，使得$\alpha_4=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3$.

要证$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$的秩为4，只要证明 $\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$线性无关.

考虑组合$x_1\alpha_1+x_2\alpha_2+x_3\alpha_3+x_4(\alpha_5-\alpha_4)=0$，即$(x_1-k_1x_4)\alpha_1+(x_2-k_2x_4)\alpha_2+(x_3-k_3x_4)\alpha_3+x_4\alpha_5=0$.

由于(III)的秩为4，所以$\alpha_1,\alpha_2,\alpha_3,\alpha_5$线性无关，所以

$$\begin{cases} x_1-k_1x_4=0 \\ x_2-k_2x_4=0 \\ x_3-k_3x_4=0 \\ x_4=0 \end{cases} \Rightarrow \begin{cases} x_1=0 \\ x_2=0 \\ x_3=0 \\ x_4=0 \end{cases}$$

所以$\alpha_1,\alpha_2,\alpha_3,\alpha_5-\alpha_4$线性无关，秩为4.

习题3.4($P_{118}-P_{119}$)

13.证明

习题3.4($P_{118}-P_{119}$)

13.证明 由矩阵的乘法知，对任意的n维列向量α，n阶方阵$\alpha\alpha^T$的列向量都是α与一个数的积，

习题3.4($P_{118}-P_{119}$)

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习题3.4($P_{118} - P_{119}$)

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(1)由于$\alpha\alpha^T$的列向量可以由α线性表出，$\beta\beta^T$的列向量可以由β线性表出，所以$\alpha\alpha^T + \beta\beta^T$可以由$\alpha, \beta$线性表出，

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所以$r(A)\leq\alpha,\beta$的秩≤ 2.

习题3.4($P_{118}-P_{119}$)

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所以$r(A)\le\alpha,\beta$的秩≤ 2.

(2)若α,β线性相关，则α,β的秩<2，从而$r(A)\le\alpha,\beta$的秩<2.

习题3.4($P_{118}-P_{119}$)

14.解

习题3.4($P_{118}-P_{119}$)

14.解 以向量X_A, X_B, X_C分别表示A, B, C三种型号混凝土中水泥、水、砂、石、灰的配比，

习题3.4($P_{118}-P_{119}$)

14.解 以向量X_A,X_B,X_C分别表示A,B,C三种型号混凝土中水泥、水、砂、石、灰的配比，即，

$$X_A=\begin{pmatrix}20\\10\\20\\10\\0\end{pmatrix},X_B=\begin{pmatrix}18\\10\\25\\5\\2\end{pmatrix},X_C=\begin{pmatrix}12\\10\\15\\15\\8\end{pmatrix}.$$

习题3.4($P_{118}-P_{119}$)

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(1)要生产五种成分为16，10，21，9，4的混凝土，设A,B,C分别需要的比例为k_1,k_2,k_3，

习题3.4($P_{118}-P_{119}$)

14.解 以向量X_A, X_B, X_C分别表示A, B, C三种型号混凝土中水泥、水、砂、石、灰的配比，即，

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(1)要生产五种成分为16，10，21，9，4的混凝土，设A, B, C分别需要的比例为k_1, k_2, k_3，则

$$k_1X_A+k_2X_B+k_3X_C=\begin{pmatrix}16\\10\\21\\9\\4\end{pmatrix}$$

习题3.4($P_{118}-P_{119}$)

$$\Leftrightarrow k_1\begin{pmatrix}20\\10\\20\\10\\0\end{pmatrix}+k_2\begin{pmatrix}18\\10\\25\\5\\2\end{pmatrix}+k_3\begin{pmatrix}12\\10\\15\\15\\8\end{pmatrix}=\begin{pmatrix}16\\10\\21\\9\\4\end{pmatrix}.$$

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对方程组的增广矩阵实施初等行变换，化为

$$\begin{pmatrix}1&0&0&0.08\\0&1&0&0.56\\0&0&1&0.36\\0&0&0&0\\0&0&0&0\end{pmatrix},$$

习题3.4($P_{118}-P_{119}$)

$$\Leftrightarrow k_1\begin{pmatrix}20\\10\\20\\10\\0\end{pmatrix}+k_2\begin{pmatrix}18\\10\\25\\5\\2\end{pmatrix}+k_3\begin{pmatrix}12\\10\\15\\15\\8\end{pmatrix}=\begin{pmatrix}16\\10\\21\\9\\4\end{pmatrix}.$$

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所以需要8%的A型，56%的B型，36%的C型.

习题3.4($P_{118}-P_{119}$)

$$\Leftrightarrow k_1\begin{pmatrix}20\\10\\20\\10\\0\end{pmatrix}+k_2\begin{pmatrix}18\\10\\25\\5\\2\end{pmatrix}+k_3\begin{pmatrix}12\\10\\15\\15\\8\end{pmatrix}=\begin{pmatrix}16\\10\\21\\9\\4\end{pmatrix}.$$

对方程组的增广矩阵实施初等行变换，化为

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所以需要8%的A型，56%的B型，36%的C型.

所以生产200kg这种混凝土，

需要16kg的A型，112kg的B型， 72kg的C型.

习题3.4($P_{118}-P_{119}$)

(2)要生产五种成分为30，57，69，7，80的混凝土，设A,B,C分别需要的比例为k_1,k_2,k_3，

习题3.4($P_{118}-P_{119}$)

(2)要生产五种成分为30，57，69，7，80的混凝土，设A,B,C分别需要的比例为k_1,k_2,k_3，则

$$k_1X_A+k_2X_B+k_3X_C=\begin{pmatrix}30\\57\\69\\7\\80\end{pmatrix}$$

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$$\Leftrightarrow k_1\begin{pmatrix}20\\10\\20\\10\\0\end{pmatrix}+k_2\begin{pmatrix}18\\10\\25\\5\\2\end{pmatrix}+k_3\begin{pmatrix}12\\10\\15\\15\\8\end{pmatrix}=\begin{pmatrix}30\\57\\69\\7\\80\end{pmatrix}.$$

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$$\Leftrightarrow k_1\begin{pmatrix}20\\10\\20\\10\\0\end{pmatrix}+k_2\begin{pmatrix}18\\10\\25\\5\\2\end{pmatrix}+k_3\begin{pmatrix}12\\10\\15\\15\\8\end{pmatrix}=\begin{pmatrix}30\\57\\69\\7\\80\end{pmatrix}.$$

对方程组的增广矩阵实施初等行变换，化为

$$\begin{pmatrix}10&10&10&57\\0&1&4&40\\0&0&5&31\\0&0&0&1\\0&0&0&0\end{pmatrix},$$

习题3.4($P_{118}-P_{119}$)

$$\Leftrightarrow k_1\begin{pmatrix}20\\10\\20\\10\\0\end{pmatrix}+k_2\begin{pmatrix}18\\10\\25\\5\\2\end{pmatrix}+k_3\begin{pmatrix}12\\10\\15\\15\\8\end{pmatrix}=\begin{pmatrix}30\\57\\69\\7\\80\end{pmatrix}.$$

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$$\begin{pmatrix}10&10&10&57\\0&1&4&40\\0&0&5&31\\0&0&0&1\\0&0&0&0\end{pmatrix}$$，显然对应的方程组无解，也就是说利用A,B,C三种型号的混凝土，不能配成客户需要的型号.

Thank you!

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